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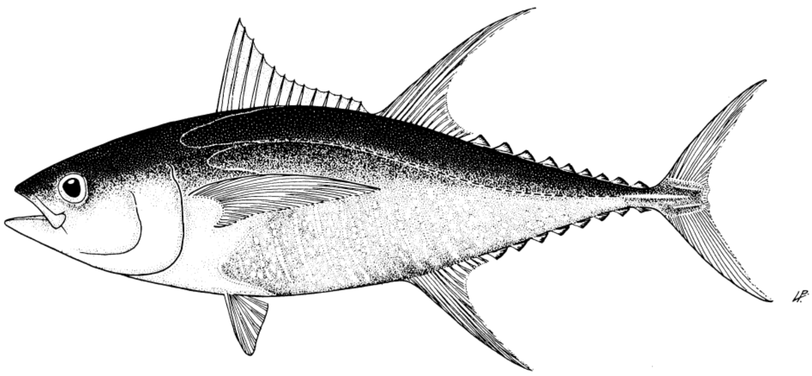
SCTB16 Working Paper

MWG–4

Theoretical consideration of estimating the varied carrying capacities



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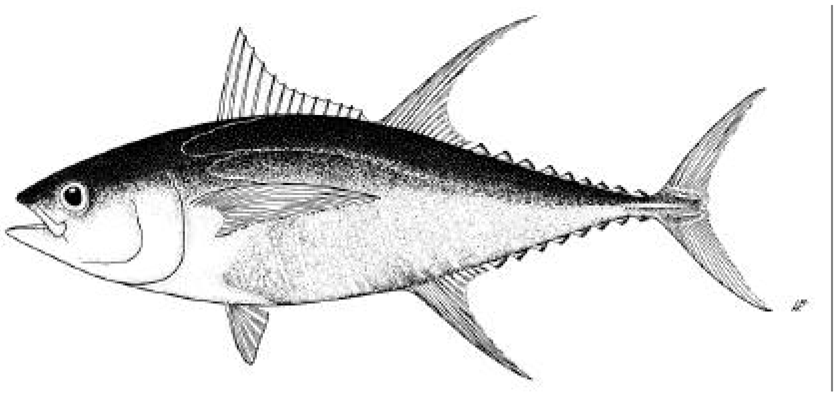

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Title:
Theoretical consideration of estimating the varied carrying capacities

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Theoretical consideration of estimating the varied carrying capacities

ABSTRACT

If only catch and effort data are available, Schaefer's model is a simple, useful and convenient method for assessing fish stocks. However, it needs generally to assume that catch is at equilibrium or not and constant carrying capacity. This paper tried to show that a generalized method can be derived from the same Schaefer's model. It can be applied to estimate the varied carrying capacities without the assumption of catch at equilibrium or not.

Keywords: Schaefer's model, generalized method, varied carrying capacity.

Introduction

Schaefer's surplus production model is a simple, useful and convenient method for assessing fish stocks. When applied in assessing fish stocks, only catch and effort data are necessary. Generally, it was based on the assumption of catch at equilibrium or non-equilibrium.

Without fishing, it was expressed as follows.

$$f = \frac{dB_t}{dt} = rB_t(1 - \frac{B_t}{K}) \qquad (1)$$

Under exploitation, it can be rewritten as follows.

$$f = \frac{dB_t}{dt} = rB_t(1 - \frac{B_t}{K}) - F_t B_t \qquad (2)$$

At equilibrium, $dB_t / dt = f = 0$, it implied that

$$U_t = qK - \frac{q^2 K}{r} X_t \qquad (3)$$

At non-equilibrium, it assumed that,

$$B_{t+1} = B_t + f = B_t + \{rB_t(1 - \frac{B_t}{K}) - F_t B_t\} \qquad (4)$$

Schnute (1977) showed that it could be transformed into following dynamic equation.

$$\ln(\frac{U_{t+1}}{U_t}) = r - \frac{r}{qK}(\frac{U_t + U_{t+1}}{2}) - q(\frac{X_t + X_{t+1}}{2}) \qquad (5)$$

On the other hand, Walters and Hilborn (1976) derived the following equation.

$$\frac{U_{t+1}}{U_t} - 1 = r - \frac{r}{qK} U_t - qX_t \qquad (6)$$

Where, *U=CPUE=catch per unit of fishing effort, r=intrinsic growth rate, B=biomass, K=carrying capacity, q= catch ability, X=fishing effort, F=qX=fishing mortality rate, t=time.* No matter which one, it needs to assume the constant carrying capacity.

As shown in equation (3), it is applicable only if CPUE decreased depending on the increasing of the fishing effort. If CPUE increased with the increasing of fishing effort, then it can not be applied in assessing fish stocks. Even it can be applied in assessing fish stocks, only MSY can be obtained. No information of the parameters *r, q and K* can be obtained.

At non-equilibrium, although the parameters *r, q, and K* can be estimated, MSY is generally unknown. If MSY is necessary in fishery management, then it needs to estimate it by other methods. Moreover, sometimes the negative parameters of biological meaningless results might be obtained. Hilborn and Walter (1992) said that this indicated model failure, that the assumption of the model were just too simple, and that by not explicitly incorporating lags to recruitment.

The problem is the assumptions of catch at equilibrium or not is absolutely necessary? Actually, the environmental conditions varied year by year. It implies the varied carrying capacities. This paper tried to suggest a generalized method of Schaefer's model in assessing fish stocks. By this method, varied carrying capacities can be obtained.

Generalized method

Under exploitation, it was conveniently setting the fishing mortality rate to be constant during a very short time period, say one year. Setting $g = r - F$ and $b = r / K$ in equation (2), then it can be rewritten as follows.

$$\frac{dB_t}{B_t dt} = r(1 - \frac{B_t}{K}) - F = g - bB_t \qquad (7)$$

Rearranged it, then equation (8) can be obtained.

$$\frac{dB_t}{B_t(g - bB_t)} = (\frac{1}{gB_t} + \frac{b}{g(g - bB_t)})dB_t = dt \qquad (8)$$

Integrating it implied

$$\int \frac{1}{gB_t} dB_t + \int \frac{b}{g(g - bB_t)} dB_t = dt$$

or

$$\frac{1}{g} \ln(\frac{B_t}{g - bB_t}) - C = t \qquad (9)$$

Here, C is the integration constant.

If $t=0$, then

$$C = \frac{1}{g} \ln(\frac{B_0}{g - bB_0}) \qquad (10)$$

Substituted it in equation (9), then

$$\frac{1}{g}\ln(\frac{B_t}{g-bB_t})-\frac{1}{g}\ln(\frac{B_0}{g-bB_0})=t$$

or

$$\ln[(\frac{B_t}{g-bB_t})/(\frac{B_0}{g-bB_0})]=\ln(\frac{(g-bB_0)B_t}{B_0(g-bB_t)}=gt$$

Hence,

$$\frac{(g-bB_0)B_t}{B_0(g-bB_t)}=e^{gt} \qquad (11)$$

Solved for *Bt*, then

$$B_t=\frac{gB_0e^{gt}}{g+bB_0(e^{gt}-1)} \qquad (12)$$

It shows the variations of biomass under exploitation.

On the other hand, the instantaneous catch, $Y_t=F_tB_t$, implied that the annual catch can be given as follows.

$$Y=\int_t^{t+1}Y_tdt=\int_t^{t+1}FB_tdt=F\int_t^{t+1}\frac{gB_0e^{gt}}{g+bB_0(e^{gt}-1)}dt \qquad (13)$$

Integrating it, the annual catch can be obtained as follows.

$$Y=F\int_t^{t+1}\frac{gB_0e^{gt}}{g+bB_0(e^{gt}-1)}dt$$
$$=F\int_t^{t+1}\frac{1}{g+bB_0(e^{gt}-1)}d(B_0e^{gt}-1)$$
$$=\frac{1}{b}F\int_t^{t+1}\frac{d[g+bB_0(e^{gt}-1)]}{g+bB_0(e^{gt}-1)}$$

or,

$$Y=\frac{F}{b}\ln(\frac{g+bB_0(e^{g(t+1)}-1)}{g+bB_0(e^{gt}-1)}) \qquad (14)$$

Equation (12) can be rearranged as follows.

$$g+bB_0(e^{gt}-1)=\frac{gB_0e^{gt}}{B_t}$$

For *t+1*, it implies that

$$g+bB_0(e^{g(t+1)}-1)=\frac{gB_0e^{g(t+1)}}{B_{t+1}}$$

Substituted both in equation (14), then

$$Y = \frac{F}{b}\ln(\frac{g + bB_0(e^{g(t+1)} - 1)}{g + bB_0(e^{gt} - 1)})$$
$$= \frac{F}{b}\ln(\frac{gB_0 e^{g(t+1)} / B_{t+1}}{gB_0 e^{gt} / B_t})$$
$$= \frac{F}{b}\ln(\frac{B_t e^g}{B_{t+1}})$$

or

$$Y = \frac{F}{b}\ln(\frac{B_t}{B_{t+1}}) + \frac{gF}{b} \qquad (15)$$

Since $g = r - F$ and $b = r / K$, then

$$Y = \frac{FK}{r}\ln(\frac{B_t}{B_{t+1}}) + \frac{(r-F)FK}{r}$$

or

$$Y = FK[1 + \frac{1}{r}\ln(\frac{B_t}{B_{t+1}}) - \frac{F}{r}] \qquad (16)$$

This is the theoretical annual catch based on the assumption of the constant catchability, intrinsic growth rate, carrying capacity and fishing mortality rate in the unit time interval, say one year. Correspondingly, B_t and B_{t+1} represent the biomass at the beginning and the end of this year, respectively. Conveniently, they can be expressed by $B_{t,b}$ and $B_{t,e}$, respectively.

Under long term exploitation, intrinsic growth rate is reasonablly assumed to be constant. Catchability is also possiblly assuming to be constant by standardized fishing effort. However, carrying capacity and fishing mortality rate fluctuated year by year. For each year, they are expressed by K_t and F_t, respectively, then equation (16) can be rewritten as follows.

$$Y_t = F_t K_t [1 + \frac{1}{r}\ln(\frac{B_{t,b}}{B_{t,e}}) - \frac{F_t}{r}] \qquad (17)$$

Equation (17) means that annual catch is determined by the fishing mortality rate, carrying capacity, intrinsic grwoth rate, and the biomass at the beginning of this year relative to the biomass at the end of this year.

Since $F_t = qX_t$ and $U_t = Y_t / X_t$, it implies that

$$U_t = qK_t[1 + \frac{1}{r}\ln(\frac{B_{t,b}}{B_{t,e}}) - \frac{q}{r}X_t] \qquad (18)$$

If it follows the assumption of catch at equilibrium, i.e., $B_{t,b} = B_{t,e}$, and constant carrying capacity, then equation (18) implies equation (3). This is the same method of catch at equilibrium.

Substituted

$$K_t = K = \text{constant}$$
$$B_{t,b} = B_t$$
$$B_{t,e} = B_{t+1}$$
$$U_t = (U_t + U_{t+1})/2$$
$$X_t = (X_t + X_{t+1})/2$$

in equation (18) and rearranged it, then equation (5) can be obtained. It is the same method suggested by Schnute (1977).

Substituted

$$K_t = K = \text{constant}$$
$$\ln(\frac{B_{t,e}}{B_{t,b}}) = \frac{U_{t+1}}{U_t} - 1 = \frac{U_{t+1} - U_t}{U_t}$$

in equation (18) and rearranged it, then equation (6) can be obtained. It is the same method suggested by Walters and Hilborn (1976).

As stated above, method-1~-3 are special case of equation (18) only. Hence, equation (18) might be a generalized method of Schaefer's model.

From equation (18), approximately, the ratio of $B_{t,b} / B_{t,e}$ can be rewritten as follows.

$$\frac{B_{t,b}}{B_{t,e}} = \frac{(U_{t-1} + U_t)/2}{(U_t + U_{t+1})/2} = \frac{U_{t-1} + U_t}{U_t + U_{t+1}} \qquad (19)$$

Substituted it in equation (18), then

$$U_t = qK_t[1 + \frac{1}{r}\ln(\frac{U_{t-1} + U_t}{U_t + U_{t+1}}) - \frac{q}{r}X_t] \qquad (20)$$

can be obtained. This is useful equation for assessing fish stocks if only catch and effort data are available.

Application of generalized method

First sight of equation (20), it seems impossible to apply it in assessing fish stocks because it contains so many parameters, *r, q, Kt, t=1,2,3…T*. If setting $a_t = B_t / K_t$ in equation (20), then

$$a_t = 1 + \frac{1}{r}\ln(\frac{U_{t-1} + U_t}{U_t + U_{t+1}}) - \frac{q}{r}X_t \qquad (21)$$

Rearranged it implied that

$$\ln(\frac{U_{t-1}+U_t}{U_t+U_{t+1}}) = r(a_t - 1) + qX_t \qquad (22)$$

Theoretically, equation (22) seems useful for estimating the parameters if only catch and effort data are available. Unfortunately, even for the simplest case of $a = a_t = \text{constant}$, only the slope *q* can be determined by the method of least squares. It needs to separate *r and a* from the intercept $r(a-1)$. It is not so easy. Even it is possible, because they are separated from the intercept $r(a-1)$, under estimation of *a* implies the over estimation of *r*, and vice sersa. Furthermore, constant *a* means the constant ratio of the biomass relative to the carrying capacity only. It is not equal to the constant carrying capacity or constant biomass.

Fortunately, equation (20) represent a series of parallel curve depending on the different intercept qK_i only. Therefore, it is conveniently setting *Ki=K=constant* to deternine the common parameters *q and r* by the method of least squares. Since *r and q* can be determined, the index a_t can be calculated year by year by equation (21). Finally, the varied carrying capacity K_t can be evaluated year by year by the definition of $a_t = B_t / K_t = U_t / qK_t$.

The relationships between a_t and K_t are rather appreciated. Experimentally, they can be expressed by $K = a + b\ a + g\ a^2$. By the definition of $a_t = B_t / K_t$, the carrying capacity of the virgin stock can be obtained by setting $a_t = 1$ in this equation.

Generally, the fluctuation of biomass can be expressed by $B_{t,e} = B_{t,b}e^{m-F}$ with the natural net production rate *m* and fishing mortality rate *F* in the unit time interval (Wang 2000, 2001, 2002). Substituted $B_{t,b} / B_{t,e} = e^{F-m}$ in equation (18) and rearranged it, then it implied that

$$a_t = 1 - \frac{m_t}{r} = \frac{r - m_t}{r} \qquad (23)$$

Clearly, it depends on the natural net production rate and intrinsic grwoth rate only. By the definition of $a_t = B_t / K_t$, it is independent on fishing mortality rate. Hence, it is mainly depending on the environmental conditions. Maybe it is an index of the fluctuated environmental condtions.

As stated above, if only catch and effort data are available, the parametrers: r, q, a_t, and K_t of each year, and the carrying capacity of the virgin stock K_v can be obtained easily. Moreover, the estimations of the index of $a_t = B_t / K_t$ are possible.

If it needs to estimate the most stable environmental conditions, then approximately the average value of a with the minimum coefficient of variance was suggested to represent the constant a of the most stable environmental conditions (Wang 2000, 2001, 2002).

When fitting catch and effort data to the equation (20) based on the assumption of constant *K*, the intercept is *qK*. This is the avarage of CPUE. Since $\bar{U} = qK$ or $\bar{B} = \bar{U}/q = K$, hence this K is the mean values of the biomass. It is not the true *K*. $B = K$ if and only if $a_t = B_t / K_t \equiv 1$. Hence, the carrying capacity of the most stable environmental conditions is estimated by $K_s = \bar{B}/a$ (Wang 2000, 2001, 2002).

Discussions and conclusions

Recently, Wang (2000,2001, 2002) tried to suggest a new method for applying the Schaefer's model in assessing fish stocks. As stated above, Wang's method seems a generalized method of Schaefer's model. The methods of equilibrium or non-equilibrium suggested by Schnute or Walters and Hilborn are special cases of the generalized method only. By generalized method, it doesn't need to assume catch at equilibrium. It doesn't need to assume the constant carrying capacity. At least, up to now, no biological impossible results are obtained. There are so many parameters can be obtained. Therefore, it seems an useful and helpful method in the field of fishery science.

The index a might be a useful tool for detecting the fluctuation of the environmental conditions. It might be helpful to know that which factor is the main factor affected the fluctuation of fish stocks. Varied carrying capacities provide us the information of the fluctuated environmental conditions.

Because the biomass is depending on the environmental conditions. Hence, carrying capacity varied year by year. Theoretically, the better environmental conditions implies the larger population size, and hence it could providing the more catch. This implies that MSY is depending on the environmental conditions. If really it is, then MSY is meaningful only if management of the environmental conditions is possible.

Theoretically, if the management of the environmental conditions is possible, then another MSY can be obtained from the generalized method. It was given as follows (Wang, unpublished).

$$MSY = \frac{arK(1-a)}{(2-a)^2}$$

$$F_{msy} = \frac{r(1-a)}{2-a}$$

$$B_{msy} = \frac{aK}{2-a}$$

if the index $a_t = B_t / K_t$ is manageable.

At present situation, controlling the environmental condition is still mission impossible. Instead of MSY, the difference $d = m - F$ might be an useful and meaningful index of fishery management. Positive of the dfference means the stocks still in increasing status. Negative of the difference means the stocks have been in decreasing status. Zero of the difference means the stable of the fish stocks. If it is target on quickly increasing of the fish stocks, then it should be target on maintaining the difference as large as possible. Contrary, if it is target on getting more catch from the stocks, then it can be target on maintaining the difference as small as possible. As stated above, this new idea is possible in fishery management because the estimation of $d = m - F$ is possible. As to the problem of how large of d is optimum, it needs more detail considerations.

Furthermore, because the estimations of *r, q, Kv, Ks, Kt and* a_t are possible, they are useful and helpful for the future research of many fields. For the same species in the same environmental conditions, it will be helpful to know the influence of the exploitation. For the same species in different environmental conditions, different parameters might imply the different adaptation to the different environmental conditions. For the different species in the same ecosystem, the different parameters might imply the different life strategy. In fishery science, commonly there are many types of fisheries targeting on many different species. It seems also helpful to know the interactions of the fisheries. By monitoring the index $d = m - F$ of each species suitably, it seems possible to establish the most appreciated ecosystem. It implies that it is possible to consider how to get MSY from some ecosystem.

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